

媒介変数で表された関数の2階導関数の求め方

$$x = f(t), \quad y = g(t) \quad (t \text{ は媒介変数})$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{g'(t)}{f'(t)}$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx} \left(\frac{dy}{dx} \right) \\ &= \frac{d}{dx} \left(\frac{g'(t)}{f'(t)} \right) \\ &= \frac{d}{dt} \left(\frac{g'(t)}{f'(t)} \right) \cdot \frac{dt}{dx} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \leftarrow \frac{g'(t)}{f'(t)} \text{ は } t \text{ の関数だから, } t \text{ でないと微分できない。} \\ &= \frac{d}{dt} \left(\frac{g'(t)}{f'(t)} \right) \cdot \frac{1}{\frac{dx}{dt}} \\ &= \frac{g''(t)f'(t) - g'(t)f''(t)}{\{f'(t)\}^2} \cdot \frac{1}{f'(t)} \\ &= \frac{g''(t)f'(t) - g'(t)f''(t)}{\{f'(t)\}^3} \end{aligned}$$